LECTURE NO 29

Topics

- Electromagnetic wave propagation:
- Wave propagation in lossy dielectrics,
- plane waves in losslessdielectrics,
- plane wave in free space, plain waves in good conductors

Wave motion occurs when a disturbance at point A, at time t_0, is related to what happens at point B, at time $t > t_0$. A wave equation, as exemplified by eqs. (9.51) and (9.52), is a partial differential equation of the second order. In one dimension, a scalar wave equation takes the form of

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0 \qquad (10.1)$$

$$\frac{d^2E_s}{dz^2} + \beta^2 E_s = 0$$

$$E^+ = Ae^{j(\omega t - \beta z)}$$

$$E^- = Be^{j(\omega t + \beta z)}$$

and

$$E = Ae^{j(\omega t - \beta z)} + Be^{j(\omega t + \beta z)} \quad (10.4c)$$

where A and B are real constants.

For the moment, let us consider the solution in eq. (10.4a). Taking the imaginary part of this equation, we have

$$E = A \sin(\omega t - \beta z) \quad (10.5)$$

Wave propagation in lossy dielectric

A lossy dielectric is a medium in which an EM wave loses power as it propagates due to poor conduction.

In other words, a lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0$, as distinct from a lossless dielectric (perfect or good dielectric) in which $\sigma = 0$.

Consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free ($\rho_v = 0$). Assuming and suppressing the time factor $e^{j\omega t}$, Maxwell's equations (see Table 9.2) become

$$\nabla \cdot \mathbf{E}_s = 0 \tag{10.11}$$

$$\nabla \cdot \mathbf{H}_s = 0 \tag{10.12}$$

$$\nabla \times \mathbf{E}_s = -j\omega\mu \mathbf{H}_s \tag{10.13}$$

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s \tag{10.14}$$

Taking the curl of both sides of eq. (10.13) gives

$$\nabla \times \nabla \times \mathbf{E}_s = -j\omega\mu \, \nabla \times \mathbf{H}_s \tag{10.15}$$

Applying the vector identity

$$\nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \tag{10.16}$$

to the left-hand side of eq. (10.15) and invoking eqs. (10.11) and (10.14), we obtain

$$\nabla (\nabla \cdot \mathbf{E}_s) - \nabla^2 \mathbf{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}_s$$

$$\swarrow 0$$

or

$$\boxed{\nabla^2 \mathbf{E}_s - \gamma^2 \mathbf{E}_s = 0} \tag{10.17}$$

where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \tag{10.18}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

In a lossless dielectric, $\sigma \ll \omega\varepsilon$. It is a special case of that in Section 10.3 except that

$$\boxed{\sigma \simeq 0, \qquad \varepsilon = \varepsilon_o\varepsilon_r, \qquad \mu = \mu_o\mu_r} \tag{10.42}$$

Substituting these into eqs. (10.23) and (10.24) gives

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon} \tag{10.43a}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta} \tag{10.43b}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^\circ \tag{10.44}$$

and thus **E** and **H** are in time phase with each other.

This is a special case of what we considered in Section 10.3. In this case,

$$\boxed{\sigma = 0, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o} \tag{10.45}$$

This may also be regarded as a special case of Section 10.4. Thus we simply replace ε by ε_o and μ by μ_o in eq. (10.43) or we substitute eq. (10.45) directly into eqs. (10.23) and (10.24). Either way, we obtain

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} \tag{10.46a}$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta} \tag{10.46b}$$